CP Violation in the Kaon System

Oct 27, 2016

Matter-Antimatter Asymmetry of the Universe

- The universe is made largely of matter with very little antimatter

$$\frac{n_B - n_{\overline{B}}}{n_\gamma} \sim 10^{-9}$$

Why is this the case?

- Matter dominance occured during early evolution of the Universe
- Assume Big Bang produces equal numbers of B and $\overline{B}$
- At high temperature, baryons in thermal equilibrium with photons

$$\gamma + \gamma \leftrightarrow p + \overline{p}$$

- Temperature and mean energy of photons decrease as Universe expands
  - Forward reaction ceases
  - Baryon density becomes low and backward reaction rare
  - Number of B and $\overline{B}$ becomes fixed

  "Big-Bang" baryogenesis
- Need a mechanism to explain the observed matter-antimatter asymmetry

The Sakharov Conditions

- Sakharov (1967) showed that 3 conditions needed for a baryon dominated Universe
  1. A least one B-number violating process so $N_B - N_{\overline{B}}$ is not constant
  2. C and CP violation (otherwise, for every reaction giving more B there would be one giving more $\overline{B}$)
  3. Deviation from thermal equilibrium (otherwise, each reaction would be balanced by inverse reaction)
- Is this possible?
  - Options exist for #1
  - #3 will occur during phase transitions as temperature falls below mass of relevant particles (bubbles)
  - #2 is the subject of today and Tuesday's lectures.
    - Today: First observation of CP violation (1964) and studies of CP violation in the neutral kaon system
    - Tuesday: Observation of CP violation in B decays (2001) and searches for CP violation outside the SM

Reminder: K^0 Mixing

- Flavor (K^0, $\overline{K}^0$) and mass eigenstates (K_S, K_L) not the same
- If CP were a good symmetry, mass eigenstates would be

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\overline{K}^0\right\rangle\right) \qquad CP\,|K_1\rangle = |K_2\rangle$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\overline{K}^0\right\rangle\right) \qquad CP\,|K_2\rangle = -\,|K_1\rangle$$

- Associating the CP states with the decays:

$$|K_1\rangle \to 2\pi$$
$$|K_2\rangle \to 3\pi$$

- However, very little phase space for 3π decay: Lifetime of $|K_2\rangle$ much longer than of $|K_1\rangle$
- Physical states called "K long" and "K short":

$$\tau(K_S) = 0.9 \times 10^{-10}\ \text{sec}$$
$$\tau(K_L) = 0.5 \times 10^{-7}\ \text{sec}$$

Discovery of CP Violation (1964)

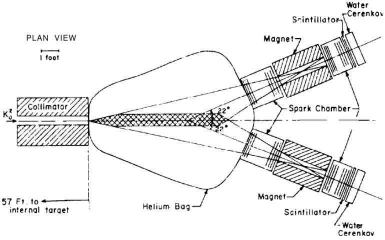


Fig. I. Plan view of the apparatus as located at the A. G. S.

(Cronin and Fitch)

- Create neutral kaon beam
- Long enough decay pipe for K_S to decay away
- Search for existence of

$$K_L \to \pi^+\pi^-$$

- Handles are:
  - Mass of $\pi^+\pi^-$ pair should be $M(K^0)$
  - Momentum of $\pi^+\pi^-$ points along beam direction

$$\left(\sum_{\pi^+\pi^-} \vec{p} \right)_\perp = 0$$

What Was Seen

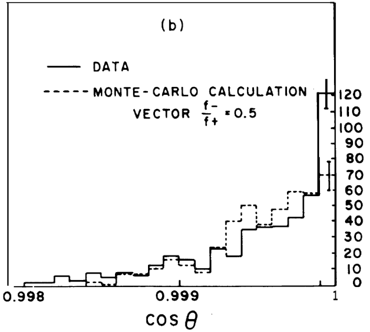
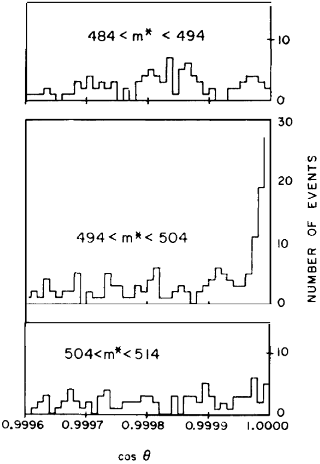


Fig. 2. Angular distributions of those events in the appropriate mass range as measured by a coarse measuring machine.



Clear evidence of $K_L \to \pi^+\pi^-$

How big is the 2π Amplitude?

- Define observed CP parameter

$$|\eta_{+-}| \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = 2.27 \times 10^{-3}$$

- Suggests CP violation is small but non-zero
- But original experiment couldn't rule out other possibilities
  - Is there a very low mass 3^{rd} particle released in the decay?
  - Are the "π"'s really pions?
- New experiment by Fitch *et al* the next year to rule these possibilities out

Are the Particles Observed in $K_L \to \pi^+\pi^-$ Really Pions?

- Neutral K beam with long decay pipe so only K_L left
- Use regenerator to create K_s. Regenerator amplitude

$$A_R = i\pi N\Lambda \left(\frac{f - \overline{f}}{k}\right)\left(i\delta + \frac{1}{2}\right)^{-1}$$

  where k wave number of incident kaon, f and $\overline{f}$ are forward scattering amplitudes, N is number density of the material, Λ is the mean decay length of the K_s, and $\delta = (M_S - M_L)/\Gamma_S$
- $K_L \to \pi^+\pi^-$ yield is proportional to $|A_R + \eta_{+-}|^2$
- Study $\pi^+\pi^-$ rate as a function of A_R
- Evidence that K_S and K_S are decaying to the same final state and have constructive interference

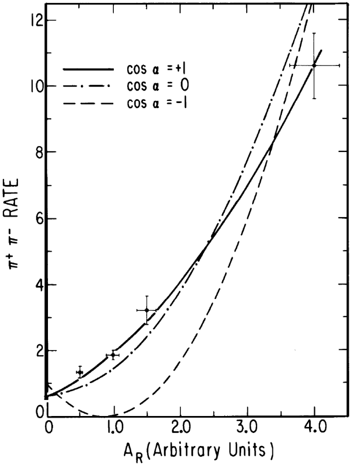


Fig. 1. Yield of $\pi^+\pi^-$ events as a function of the diffuse regenerator amplitude. The three curves correspond to the three stated values of the phase between the regeneration amplitude A_r and the CP violating amplitude η_{+-}.

More Evidence for CP Violation

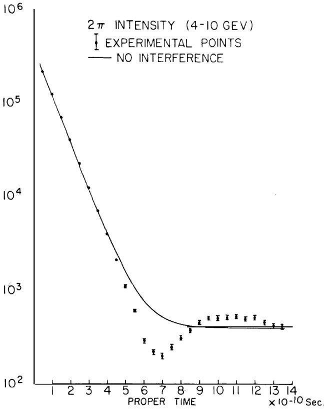


Fig. 2. Yield of $\pi^+\pi^-$ events as a function of proper time downstream from an 81 cm carbon regenerator placed in a K_L beam.

- Pick Regenerator Thickness to Give Equal K_S and K_L Populations

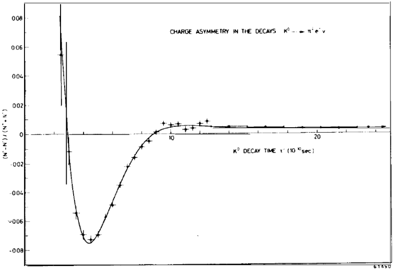


Fig. 3. Time dependence of the charge asymmetry of semileptonic decays.

- Clear Evidence of CP Violation in semileptonic decays as well

$$\delta_\ell = \frac{\Gamma(K_L \to \pi^- \ell^+ \nu_\ell) - \Gamma(K_L \to \pi^+ \ell^- \overline{\nu}_\ell)}{\Gamma(K_L \to \pi^- \ell^+ \nu_\ell) + \Gamma(K_L \to \pi^+ \ell^- \overline{\nu}_\ell)} = 3.3 \times 10^{-3}$$

One Additional Observable: η_{00}

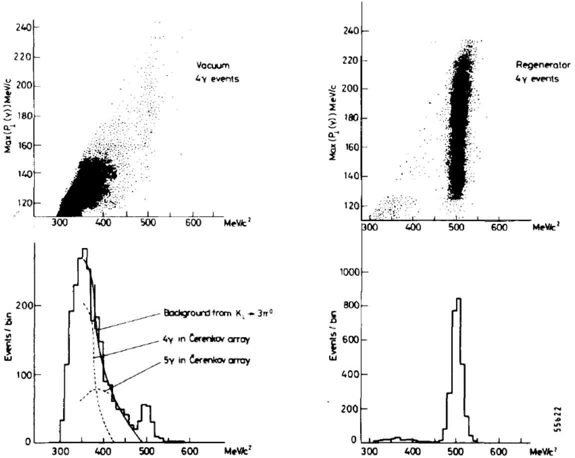
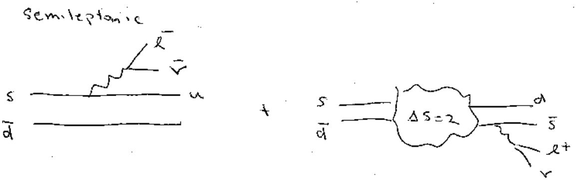
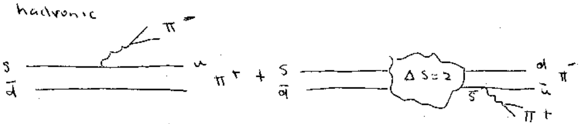


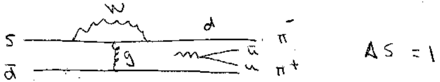
Fig. 4. Distributions of reconstructed $K_L \to \pi^0\pi^0$ events, and regenerated $K_s \to \pi^0\pi^0$ events

$$|\eta_{00}| = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = 2.2 \times 10^{-3}$$

Characterizing CP Violation (I)







- Mixing diagrams may contain CP-violating terms. [They do in the SM (CKM)]

  These diagrams have $\Delta S = 2$
- Both semi-leptonic and hadronic decays can have $\Delta S = 2$
- There may also be diagrams with CP violating terms that have nothing to do with mixing
- These occur via WI because strangeness can't be conserved. We have

  $\Delta S = 1$ (Example shown to left)
- Only hadronic decays can have $\Delta S = 1$

Characterizing CP Violation (II)

- $\Delta S = 2$ required for semi-leptonic decays but both $\Delta S = 2$ and $\Delta S = 1$ possible for hadronic decays
- Fact that δ, η_{00} and η_{+-} all have similar size indicates that $\Delta S = 2$ dominates
- Express CP violation in the mixing can be described by saying K_L has a bit of $|K_1\rangle$ and K_S has a bit of $|K_2\rangle$

$$
\begin{aligned}
|K_S\rangle &= \frac{(|K_1\rangle + \epsilon\,|K_2\rangle)}{\sqrt{1+|\epsilon|^2}} \\
|K_L\rangle &= \frac{(|K_2\rangle + \epsilon\,|K_1\rangle)}{\sqrt{1+|\epsilon|^2}}
\end{aligned}
$$

  where the normalization is good to order ϵ
- Note: $|K_S\rangle$ and $|K_L\rangle$ are NOT orthoginal
- Expressing above in terms of K^0 and $\overline{K}^0$:

$$
\begin{aligned}
|K_S\rangle &= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\epsilon|^2}}\left((1+\epsilon)\left|K^0\right\rangle + (1-\epsilon)\left|\overline{K}^0\right\rangle\right) \\
|K_L\rangle &= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\epsilon|^2}}\left((1+\epsilon)\left|K^0\right\rangle - (1-\epsilon)\left|\overline{K}^0\right\rangle\right)
\end{aligned}
$$

CP From Mixing Vs Direct CP

- We saw last time

$$i\frac{d\psi}{dt} = \begin{pmatrix} M - i\frac{i}{2}\Gamma/2 & M_{12} - i\frac{i}{2}\Gamma_{12}/2 \\ M^*{}_{12} - i\frac{i}{2}\Gamma^*{}_{12}/2 & M - i\frac{i}{2}\Gamma/2 \end{pmatrix}\psi$$

- If we write $\delta m = \delta m_R + i\delta m_I$ can show

$$\epsilon = \frac{i\delta m_I}{m_L - m_S + i\Gamma_S/2}$$

- You will show on HW that

$$\delta_\ell = 2\mathrm{Re}\ \epsilon$$

- If direct CP ($\Delta S = 1$) will need one additional parameter (called ϵ').
  - In K system, this is small, even when compared to ϵ

A General Description of CP Violation in K^0s

- Decompose 2π state into $I = 0$ and $I = 2$ (no $I = 1$ since $L = 0$ and Bose Statistics)
- Can define 4 Amplitudes:

$$\begin{aligned}
\langle 2\pi, I = 0| H_{wk} \left|K^0\right\rangle &= A_0 \\
\langle 2\pi, I = 0| H_{wk} \left|\overline{K}^0\right\rangle &= -A^*{}_0 \\
\langle 2\pi, I = 2| H_{wk} \left|K^0\right\rangle &= A_2 \\
\langle 2\pi, I = 2| H_{wk} \left|\overline{K}^0\right\rangle &= -A^*{}_2
\end{aligned}$$

- Three physical measurements

$$\begin{aligned}
\eta_{+-} &= \frac{\left\langle \pi^+\pi^- \right| H_{wk} \left|K_L\right\rangle}{\left\langle \pi^+\pi^- \right| H_{wk} \left|K_S\right\rangle} \\
\eta_{00} &= \frac{\left\langle \pi^0\pi^0 \right| H_{wk} \left|K_L\right\rangle}{\left\langle \pi^0\pi^0 \right| H_{wk} \left|K_S\right\rangle} \\
\delta_\ell &= \frac{\Gamma(K_L \to \pi^-\ell^+\nu_\ell) - \Gamma(K_L \to \pi^+\ell^-\overline{\nu}_\ell)}{\Gamma(K_L \to \pi^-\ell^+\nu_\ell) + \Gamma(K_L \to \pi^+\ell^-\overline{\nu}_\ell)}
\end{aligned}$$

- Now break into $I = 0$ and $I = 2$

Isospin Decomposition

- Using Clebsh-Gordon coeff:

$$\begin{aligned}\left|\pi^+\pi^-\right\rangle^{symm} &= \frac{1}{\sqrt{2}}\left|\pi^+\pi^- + \pi^-\pi^+\right\rangle \\ &= \frac{1}{\sqrt{3}}\left(|I=2\rangle + \sqrt{2}\,|I=0\rangle\right) \\ \left|\pi^0\pi^0\right\rangle^{symm} &= \frac{1}{\sqrt{3}}\left(\sqrt{2}\,|I=2\rangle + |I=0\rangle\right)\end{aligned}$$

- In above have ignored final state interaction. These add a "strong phase" which is different for $I = 0$ and $I = 2$
- Define

$$\begin{aligned}A_0 e^{i\delta_0} &= \langle I=0|\, H_{wk} \left|K^0\right\rangle \\ A_2 e^{i\delta_2} &= \langle I=0|\, H_{wk} \left|K^0\right\rangle \\ A_0^* e^{i\delta_0} &= \langle I=2|\, H_{wk} \left|\overline{K}^0\right\rangle \\ A_2^* e^{i\delta_2} &= \langle I=2|\, H_{wk} \left|\overline{K}^0\right\rangle\end{aligned}$$

- By convention take A_0 as real

Completing the Math

- We find:

$$
\begin{aligned}
\left\langle \pi^+\pi^- \right| H_{wk} \left| K_L \right\rangle &= \sqrt{2/3}e^{i\delta_2}(\epsilon Re\ A_2 + iIm\ A_2) + 2\sqrt{1/3}e^{i\delta_0}(\epsilon Re\ A_0 + iIm\ A_0) \\
\left\langle \pi^0\pi^0 \right| H_{wk} \left| K_L \right\rangle &= 2\sqrt{1/3}e^{i\delta_2}(\epsilon Re\ A_2 + iIm\ A_2) - \sqrt{2/3}e^{i\delta_0}(\epsilon Re\ A_0 + iIm\ A_0) \\
\left\langle \pi^+\pi^- \right| H_{wk} \left| K_S \right\rangle &= \sqrt{2/3}(e^{i\delta_2}Re\ A_2 + \sqrt{2}e^{i\delta_0}Re\ A_0) \\
\left\langle \pi^0\pi^0 \right| H_{wk} \left| K_S \right\rangle &= \sqrt{2/3}(\sqrt{2}e^{i\delta_2}Re\ A_2 - \sqrt{2}e^{i\delta_0}Re\ A_0)
\end{aligned}
$$

- By convention A_0 is real so

$$
\begin{aligned}
\eta_{+-} &= \epsilon + \epsilon' \\
\eta_{00} &= \epsilon - 2\epsilon' \\
\epsilon' &= \frac{1}{\sqrt{2}}\frac{Im\ A_2}{A_0}\exp(i\pi/2 - i\delta_0 + i\delta_2)
\end{aligned}
$$

Measurement of ϵ'/ϵ

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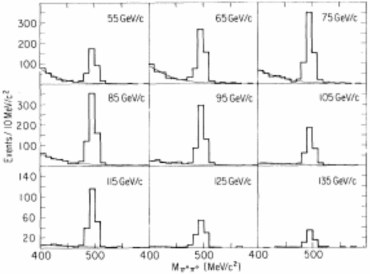


FIG. 2. Invariant-mass distributions for $K_L \to 2\pi^0$ candidates with $P_T^2 < 2500$ (MeV/c)^2. A fit to the background is superimposed.

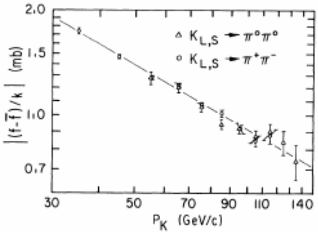
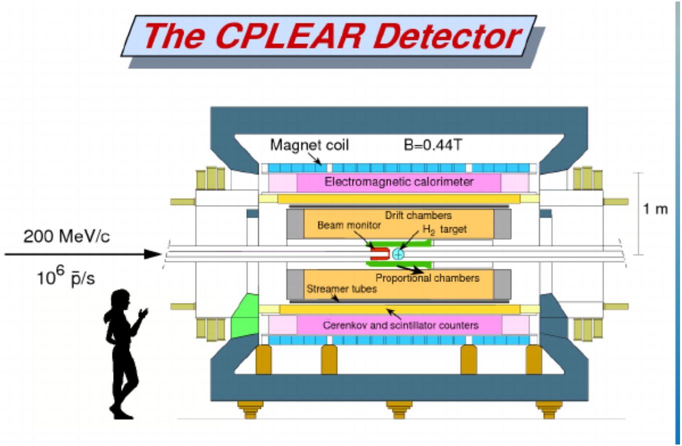


FIG. 3. $|(f-\bar{f})/k|$ for carbon vs momentum from $\pi^+\pi^-$ and $\pi^0\pi^0$ samples. The best power-law fit is superimposed. Were $\epsilon'/\epsilon = 0.01$, the neutral points would lie about 3% above the charged points.

- Must have precision to determine that η_{00} and η_{+-} have different values

2014 PDG Average: $Re(\epsilon'/\epsilon) = (1.66 \pm 0.23) \times 10^{-3}$

A More Recent Kaon CP Experiment: CPLear

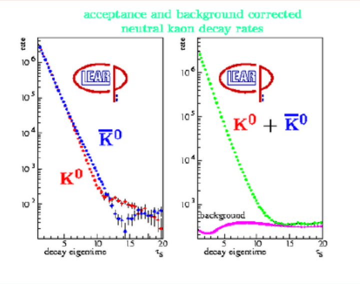


- Data taking 1990-1996 at CERN
- Anti-protons stopped in hydrogen target

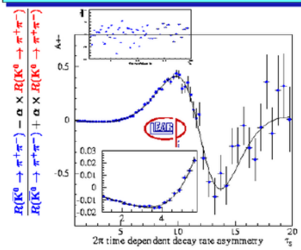
$$p\overline{p} \to K^{\pm}\pi^{\mp}K^{0}$$

- Strangeness of neutral kaon at production tagged by charge of charged kaon

CPLear Measurement of η_{+-}



Time dependent decay rate asymmetry



$$A_{+-}(\tau) = -\frac{2|\eta_{+-}| e^{\frac{1}{2}(1/\tau_S - 1/\tau_L)\tau} \cos(\Delta m \cdot \tau - \varphi_{+-})}{1 + |\eta_{+-}|^2 e^{(1/\tau_S - 1/\tau_L)\tau}}$$

$$|\eta_{+-}| = (2.264 \pm 0.023_{sta.} \pm 0.026_{sys.} \pm 0.007_{\tau_S}) \times 10^{-3}$$
$$\varphi_{+-} = 43.19^\circ \pm 0.53^\circ_{sta.} \pm 0.28^\circ_{sys.} \pm 0.42^\circ_{\Delta m}$$

with $\Delta m = (530.1 \pm 1.4) \times 10^7 \hbar s^{-1}$ PDG '98

published in Phys. Lett. B 458 (1999) 545

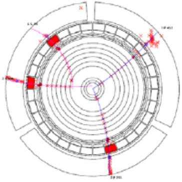
- α is a free parameter in the fit, $\alpha = \frac{\varepsilon(K^+)}{\varepsilon(K^-)}[1 + 4\mathbb{R}(\varepsilon_T + \delta)]$ used as rate normalization in other decay channels

$$A_{2\pi} = \frac{R(\overline{K}^0 \to \pi\pi)(\tau) - \alpha \times R(K^0 \to \pi\pi)(\tau)}{R(\overline{K}^0 \to \pi\pi)(\tau) + \alpha \times R(K^0 \to \pi\pi)(\tau)} = -2|\eta_{\pi\pi}| \cos(\Delta m\tau - \varphi_{\pi\pi}) \frac{e^{\frac{1}{2}(\Gamma_S - \Gamma_L)\tau}}{1 + |\eta_{\pi\pi}|^2 e^{(\Gamma_S - \Gamma_L)\tau}}$$

With Δm free in the fit, not assuming CPT,
$\Delta m = (524.0 \pm 4.4 \pm 3.3) \times 10^7 \hbar s^{-1}$

CPLear Measurement of δ

Analysis of $K^0 \to \pi^{\mp} e^{\pm} \nu$



- kinematical constraints
- electron identification based on:
- dE/dx in the scintillators,
- number of photo-electrons in the Čerenkov,
- number of hits in the calorimeter

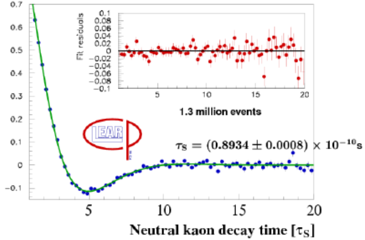
Precise measurement of the oscillation frequency Δm (setting $\Im(x_-)$=0) :

Δm and $\Im(x_-)$ are strongly correlated, >0.99.
With $\Delta m = (530.1 \pm 1.4) \times 10^7 \hbar s^{-1}$ obtain
$\Im(x_-) = (-0.8 \pm 3.5) \times 10^{-3}$

$K_L - K_S$ Mass Difference

$$A_{\Delta m} = \frac{N_{K^0 \leftarrow K^0, \overline{K}^0 \leftarrow \overline{K}^0} - N_{\overline{K}^0 \leftarrow K^0, K^0 \leftarrow \overline{K}^0}}{N_{K^0 \leftarrow K^0, \overline{K}^0 \leftarrow \overline{K}^0} + N_{\overline{K}^0 \leftarrow K^0, K^0 \leftarrow \overline{K}^0}}$$

$$= 2\frac{e^{-\overline{\Gamma}\tau}\cos\Delta m\tau + 2\Im(x_-)e^{-\overline{\Gamma}\tau}\sin\Delta m\tau}{[1+2\Re(x_+)]\,e^{-\Gamma_S\tau} + [1-2\Re(x_+)]\,e^{-\Gamma_L\tau}}$$



$$\Delta m = (529.5 \pm 2.0_{stat.} \pm 0.3_{syst.}) \times 10^7 \hbar s^{-1}$$

$$\Delta m = (348.5 \pm 1.3) \times 10^{-9}\ eV/c^2$$

$\Delta S = \Delta Q$ violating decays or wrong tagging:

$$\Re e\, x_+ = (-1.8 \pm 4.1_{stat.} \pm 4.5_{syst.}) \times 10^{-3}$$

Best single measurements: Phys.Lett. B444 (1998) 38

A Modern Treatment of CP Violation

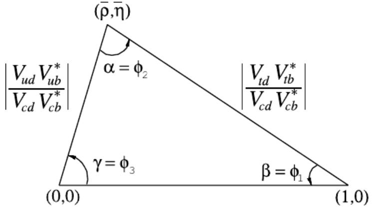


Figure 12.1: Sketch of the unitarity triangle.

$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$
$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$
$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

- CKM Matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & Vus & V_{ub} \\ V_{cd} & Vcs & V_{cb} \\ V_{td} & Vts & V_{tb} \end{pmatrix}$$

- Wolfenstein parameterization:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- Unitary Triangle:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

Classifying CP Violating Effects

- CP Violation in Decays

$$\Gamma(P^0 \to f) \neq \Gamma(\overline{P}^0 \to f)$$

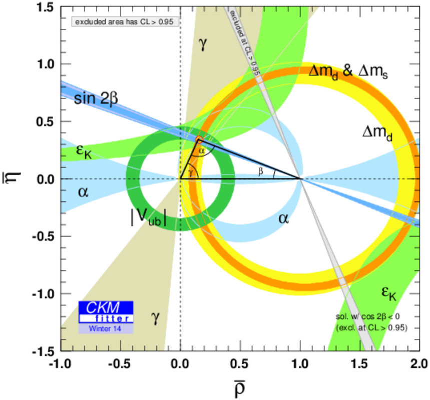
- CP Violation in Mixing

$$Prob(P^0 \to \overline{P}^0) \neq Prob(\overline{P}^0 \to P^0)$$

- CP Violation in Interference
  - Time dependent asymetry dependent on fraction of P^0 at time t

B-decays will provide a rich laboratory for studying all three of these

Combined Results of All Experiments



- Unlike K system, B decays provide MANY ways to measure CP violation
- Want to determine if all consistent with single value of (ρ, η)
- Pick measurements where theoretical uncertainties under control

  This will be the topic of next Tuesday's class